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**GEOMETRICAL OPTICS AND THE DESIGN OF  
FEED-MOTION-SCAN, PARALLEL-PLATE,  
RADAR ANTENNAS**

**by**

**James M. Meek**

**September 1970**



**U.S. ARMY MATERIEL COMMAND,  
HARRY DIAMOND LABORATORIES  
WASHINGTON, D.C. 20438**

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### ABSTRACT

Design configurations of nonoscillatory feed, line-scan, parallel-plate radar antennas have been explored. Some historically important designs are described as a basis for understanding several new, proposed antenna geometries. It is shown that the continuous-motion scanner developed by Lewis can be combined with many parallel-plate lenses and reflectors in various geometric layouts to obtain useful microwave antennas. Designs that would normally produce good radiation characteristics but would be too cumbersome become compact and useable by combining the continuous-motion scanner and the cut, rolled, and folded sectoral horn-lens assemblies.

The continuous-motion scanner can be combined with dielectric, metal-plate, and geodesic lenses. Multiple linear and nonlinear reflectors may be installed in the sectoral horn, and planar reflectors may be used in some geodesic lenses to achieve compactness. Some of these "oblique" layouts are shown to be analogous to optical telescopes. Methods of reducing antenna size by folding over or rolling up flat portions of the sectoral horn are explained.

Optical laws are shown to apply to the geometrical design of parallel-plate antennas and lenses. Coma and spherical aberration are defined, and procedures for designing parallel-plate radar antennas are proposed. Choice of optical and radar system parameters and the interrelationship of such parameters as scan angle, focal length, beamwidth, scan rate, and physical dimensions are discussed. Fermat's extremum principle is defined, and some fundamental criteria for designing geodesic lenses are outlined. Also described in this report are the oblique, multiple-reflector antenna designs conceived at HDL by the author. The versatility and compactness of these designs when combined with the continuous-motion scanner are confirmed. The design areas that require further investigation are mentioned.

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## 1. BACKGROUND

The antenna designs discussed herein are used in fire control systems and are also useful in other applications requiring range and angle detection of spacial objects. Early radar antennas utilized motion of the entire antenna to achieve volume coverage. By 1944, several designs had appeared that provide line scanning by oscillatory-feed motion; by 1947, both Schneider<sup>1</sup> and Lewis<sup>2</sup> had described nonoscillatory-feed, line-scanning systems. Their designs employ a large, trapezoidal, parallel-plate, sectoral horn, operable in the transverse electric and magnetic (TEM) wave mode. The smaller input end of the trapezoidal horn is formed annularly, so that a small feed horn can be continuously rotated at the annular orifice. ~~Rotation of the feed horn can be continuously rotated at the annular orifice.~~ Rotation of the feed horn at the input orifice results in repetitive angular displacement of the beam emanating from the large output aperture, which scans a sectoral volume of space. In operation, the angular position of a target is determined as a function of the angular position of the horn, the horn angle being proportional to the beam-swept angle. Usually, electronic tracking circuitry is employed to measure the angle as a function of elapsed time between a scan start position pulse provided by a motion transducer on the scanner and the "center" of the energy reflected from the target. Range is determined by the time required for the microwave pulses to propagate to the tracked object and back to the receiver.

Antennas that produce a scanning beam by feed motion and without the motion of either the reflector or the external portion of the feed are defined as electrically scanning antennas;<sup>3</sup> they are, however, often referred to as electromechanical scanners. Experience has shown that scan repetition rates up to 30 Hz or greater can be obtained conveniently with such scanners, having peak transmission power in the megawatt (MW) region. Since the output beam is line scanned, two dimensions are measurable with a single antenna within the space sector—for example, range and azimuth angle. To make cross-angle measurements such as elevation, an additional orthogonally positioned antenna may be employed.

To form the characteristic fan beam (a pencil beam may also be generated), it is necessary to employ a phase-correcting lens or reflector at or near the output of the large sectoral horn. Lenses of the geodesic, metal-plate, and dielectric types are employed in the horn to provide the collimation in the narrow-angle dimension of the fan beam. An external reflector may be used to provide the desired amount of collimation in the broad-angle dimension.

<sup>1</sup>"Radar," Proc of IRE, E. G. Schneider, Aug 1946.

<sup>2</sup>Patent application SH 789, 602, W. D. Lewis, 4 Dec 1947.

<sup>3</sup>"Radar Scanners and Radomes," MIT Rad Lab, vol 26, Cady et al, McGraw Hill Book Company, 1948.

Variations in the method of achieving nonoscillatory scanning have been constructed successfully. A notable variation is found in the microwave analog of the Schwarzschild telescope (sect 4.2.2). In this design, the parallel-plate region of the sectoral horn is folded and bulged and the feed arc located so that it forms a segment of the feed horn-scan circle, permitting continuous rotation of one or more feed horns over the arc. The location and shape of the folds are such that they result in a compact antenna box and also serve the function of a geodesic lens. In this design, the scanner does not achieve the compactness of the Lewis scanner.

It is characteristic of the various parallel-plate antennas that they are folded, rolled up, or contain an internal reflector, resulting in low bulk and weight.

Recent work at HDL<sup>1</sup> has indicated that a number of interesting and advantageous new antenna designs can be developed, based on oblique, parallel-plate layouts employing multiple reflectors. These designs, described in sections 4.5 and 4.6, utilize various geodesic lenses, simple parabolic-line reflectors, and multiple conic-section-line reflectors.

Spherical aberration and coma are two defects of parallel-plate antennas that tend to limit radar resolution. All lenses and reflectors described are afflicted by one or the other or both of these defects. If not sufficiently controlled, they may result in degradation of the far-field antenna beam pattern and corresponding impairment of radar resolution.

This report considers the geometrical, optical, and physical construction of parallel-plate antennas with the objective of describing antenna systems that are electrically and electronically functional. The optical treatment of these antennas often permits satisfactory initial designs to be derived. Ultimately, prototype or model antennas should be tested for electronic and radiation performance before determining the final designs.

The following sections describe the optical properties of parallel-plate radar antennas, existing antenna designs and prior art pertinent to the proposed new antenna designs, and details of the geometrical development of the new designs.

## 2. GEOMETRICAL OPTICS

If a flat, trapezoidal, sectoral horn is imagined to be constructed of parallel sheets of electrically conducting material spaced one-half wavelength or less apart and if a microwave point source is positioned

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<sup>1</sup>Laboratory Workbook HDL-457, 18 Mar 1968, pp. 22, 24, 25, 26, and following.

at the narrow edge or mouth of the trapezoid, circular waves will propagate through the horn at the free-space velocity of light. Linear reflectors may be placed between the parallel plates to change the direction of propagation without changing the speed of propagation or the ultimate shape of the wavefront. Furthermore, the horn may be rolled up or curved with discretion into developable geometric shapes (cones and cylinders); and, so long as the input and output edges are not distorted, the input-output optical characteristics will remain unaltered. On the other hand, if the reflectors placed between the parallel, flat sheets are curved or if the parallel sheets are stretched or bulged without changing the original spacing between sheets, the shape of the wave will be altered and a focusing effect can be obtained because of the altered optical paths. As an alternative, a plastic or dielectric lens may be installed within the trapezoid to provide focussing as a result of the change in propagation velocity within the dielectric medium.

Kunz<sup>1</sup> states that the whole of geometrical optics may be summarized by Fermat's principle—that when light propagates by any path from a point in a space or medium to any other point, the time required in its passage is either a minimum or a maximum—an extremum as compared with other adjacent paths. An alternate statement of Fermat's principle is as follows: the mathematical product of the geometric path length and the index of refraction is an extremum for the optical (ray) path.

Before Fermat, the Greek mathematician, Hero, deduced the law of reflection on the theory that light will travel from one point to another via a reflector by the shortest route. In 1657, P. de Fermat deduced the principle, which bears his name, in a successful attempt to define the law of refraction. Later, Huygens found that the extremum principle holds for curvilinear as well as rectilinear motions of light and for variable refractive indices. In 1696, John Bernoulli, in his work in the field of scientific mechanics, imagined the motion of a falling body to be equivalent to the motion of a ray of light and successfully determined the extremal curve (trajectory) between two horizontally displaced points.<sup>2</sup> Later researchers expanded upon these ideas in optics, geometry, and mechanics, culminating in the development of the calculus of variations by Euler, Lagrange, and others.

Listed below are optical laws derived from Fermat's principle that are useful in designing or evaluating parallel-plate antennas and geodesic lenses.

---

<sup>1</sup> "Propagation of Microwaves," K. S. Kunz, J. Appl. Opt., vol 25, no. 5, May 1954.

<sup>2</sup> "The Science of Mechanics," ch IV, E. Mach, English translation.



<u>Law or principle</u>	<u>Formulation or description</u>
Fermat's principle	$\text{Extremum} = \int n ds = c \int \frac{ds}{v} = c \int dt$ where $v$ = velocity in a medium
Law of reflection	$\theta_1 = \theta_2$ (angle of incidence equals angle of reflection)
Law of refraction (Snell's law)	$n_1 \sin \theta_1 = n_2 \sin \theta_2$
Theorem of Clairaut	$r \sin \alpha = h$ or $\sin \alpha = h/r$ The direction ( $\sin \alpha$ ) of a geodesic line relative to a polar meridian on a surface of revolution equals the ratio of the radius of closest approach of the geodesic to the axis of revolution, $h$ , and the radius to the intersection of the two lines, $r$ , taken normal to the axis of revolution.
Law of Malus and Dupin	Reflections and refractions of light may change the shape of the wave, but the optical rays will remain orthogonal to the tangents (orthotonic) at the wavefront.

One approach to the geodesic lens design problem is to use the optical principles to compute geodesic-path or wave-phase errors through a lens of predetermined mathematical shape.<sup>1</sup> Another approach that has been used for surface-of-revolution lenses is to build up by a series of computations a configuration approximated by a stack of elemental conical frusta.<sup>2</sup>

Although geometrical optics or ray theory has been found to facilitate evaluation of lenses and reflectors for radar antennas, Levi<sup>3</sup> points out that the method is limited wherever diffraction effects predominate, and precise results can be obtained then only by using the wave optical method.

### 3. DESIGN PROCEDURES

The steps and considerations involved in the initial design of a scanning, parallel-plate antenna are explored next. The output angular beamwidth for a given antenna can be determined from

<sup>1</sup>"Parallel Plate Optics for Rapid Scanning," S. B. Myers, Journal of Applied Physics, Vol. 18, Feb 1947.

<sup>2</sup>"The Geodesic Luneburg Lens, R. C. Johnson," The Microwave Journal, August 1962, pp. 76-85.

<sup>3</sup>"Applied Optics," vol 1, Leo Levi, J. Wiley & Sons, 1968.

$$\theta = \frac{K\lambda}{D}$$

where

$\lambda$  = operating wavelength,

D = length of aperture in cases of linear apertures, and

K = a coefficient pertaining to the type of antenna.

Angle  $\theta$  is the far-field beamwidth beyond about  $2D^2/\lambda$  in range from the location of the aperture<sup>1</sup>. The phase error at the output is usually considered satisfactory if it does not vary in excess of  $\lambda/16$  across the aperture.

Once the desired beamwidth and operating wavelength are chosen, the length of the output aperture may be determined, assuming the coefficient K is known.

In the parallel-plate antenna, the focal length and scan angle must also be selected before the overall antenna design configuration can be determined. Selection of the scan angle is based on the system requirements. Some lenses provide good resolution for small scan angles, but coma deterioration is excessive for large angles. If a wide scan angle is desired, coma may be eliminated by employing certain surface-of-revolution lenses, such as the Rinehart-Luneburg "tin-hat" lens (sect 4.4.1); for small scan angles where coma is not of great concern, a lens having minimum spherical aberration may be selected. For electrical scanning using the rotating feed horn as described in section 1, the physical size of the scanner head is a consideration. The scan arc length, and consequently the scanner head diameter, increases with increasing scan angles and focal lengths. Scanner centrifugal and gyroscopic effects would, at high angular velocities, impose an upper limit on the radar scan rate. Often the focal length of parallel-plate antennas is determined simply to be compatible with the other parameters—that is, a convenient value is chosen so that it fits the other geometrical parameters. Recent evaluations at HDL have disclosed that an additional restriction is imposed on the focal length by the number of internal reflections chosen and the method of folding or rolling up the trapezoidal horn. For example, when two oblique reflectors are used, obscuration or masking problems may preclude the short focal lengths that may be obtained in single-reflector designs. Conversely, the long focal length may be preferred for improved optical characteristics of the lens or for other reasons.

As may be surmised, there are usually many conventional combinations of lenses, reflectors, and parallel-plate geometries that will, to some degree, provide desired performance. A logical procedure under these circumstances is to select the simplest combination that gives good system performance for the important operational circumstances.

<sup>1</sup>"Modern Radar," R. S. Berkowitz, J. Wiley & Sons, p. 381, 1965.

Finally, the rotating scanner feed must be designed to illuminate the output reflector or lens properly. The feed is usually taken as a directional point source located at the annular inlet aperture, generating circular waves directed toward the optical center of the lens or reflector for all angles of scan. Details of feed-horn design are reported by MIT<sup>1</sup> and Fradin.<sup>2</sup>

#### 4. ANTENNA DESIGNS

##### 4.1 Telescopic Analogies

The various conventional parallel-plate fold and reflector arrangements may be compared with conventional telescope layouts. Sections through the Herschelian, Newtonian, and Cassegrainian telescopes are depicted in figure 1. The Lewis antenna described in section 4.2.1 has the single linear oblique reflector (in flat-plate form), which in some respects is the 2-D counterpart of the Newtonian telescope. The curved reflector in the telescope is the counterpart of the antenna output lens. The features of the Cassegrainian 3-D antenna layout correspond to those of the Cassegrainian telescope, whereas the parallel-plate Schwarzschild antenna achieves a resemblance by folding in layers at intervals to provide the compact design. These examples serve to illustrate the similarity of the geometrical techniques used in parallel-plate antennas and optical instruments to achieve compactness while retaining desirable optical and radiation characteristics.

##### 4.2 Typical Existing Designs

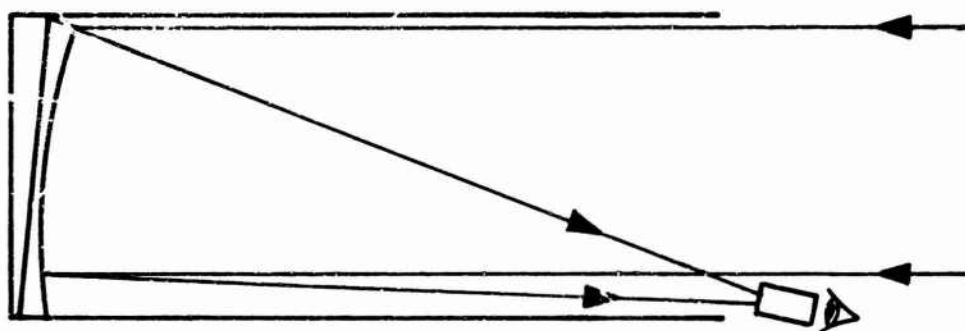
The Lewis, Cassegrainian, and Schwarzschild antennas described in sections 4.2.1 and 4.2.2 are typical existing designs. Original features of these designs have been combined with novel, multiple-reflector geometries and various lenses to achieve interesting new designs (sect. 4.6).

##### 4.2.1 Lewis Antenna

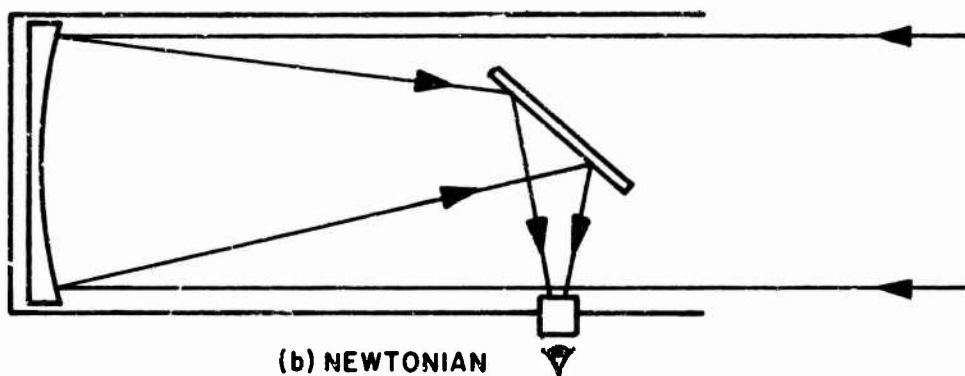
Figure 2 shows a plan view or "flat-plate" layout of the Lewis antenna before rolling up. The parallel, conducting sheets are separated  $\lambda/2$  or less for TEM mode operation. The median plane of the parallel sheets is visualized as being folded upon itself at about a 45-deg angle as indicated in the figure. Next, the parallel plates are rolled into a cone at the inlet aperture so that the scan arc becomes a circle. The rotating head, driven by an electric motor and containing the feed horn, is placed adjacent to the annular aperture. Rolling of the conic shape from the flat is effected in the median plane without "stretching" the hypothetical surface and,

<sup>1</sup>MIT Rad Lab Report 655, Jan 1945.

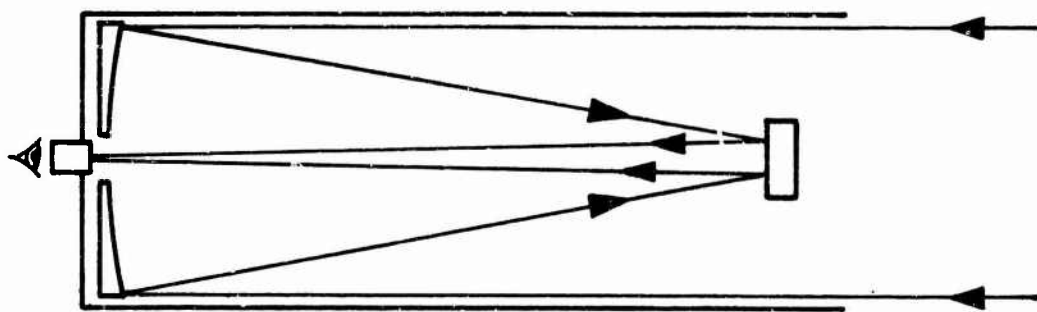
<sup>2</sup>"Microwave Antennas," A. Z. Fradin, Pergamon Press, 1961.



(a) HERSCHELIAN



(b) NEWTONIAN



(c) CASSEGRAINIAN

Figure 1. Telescope layouts.

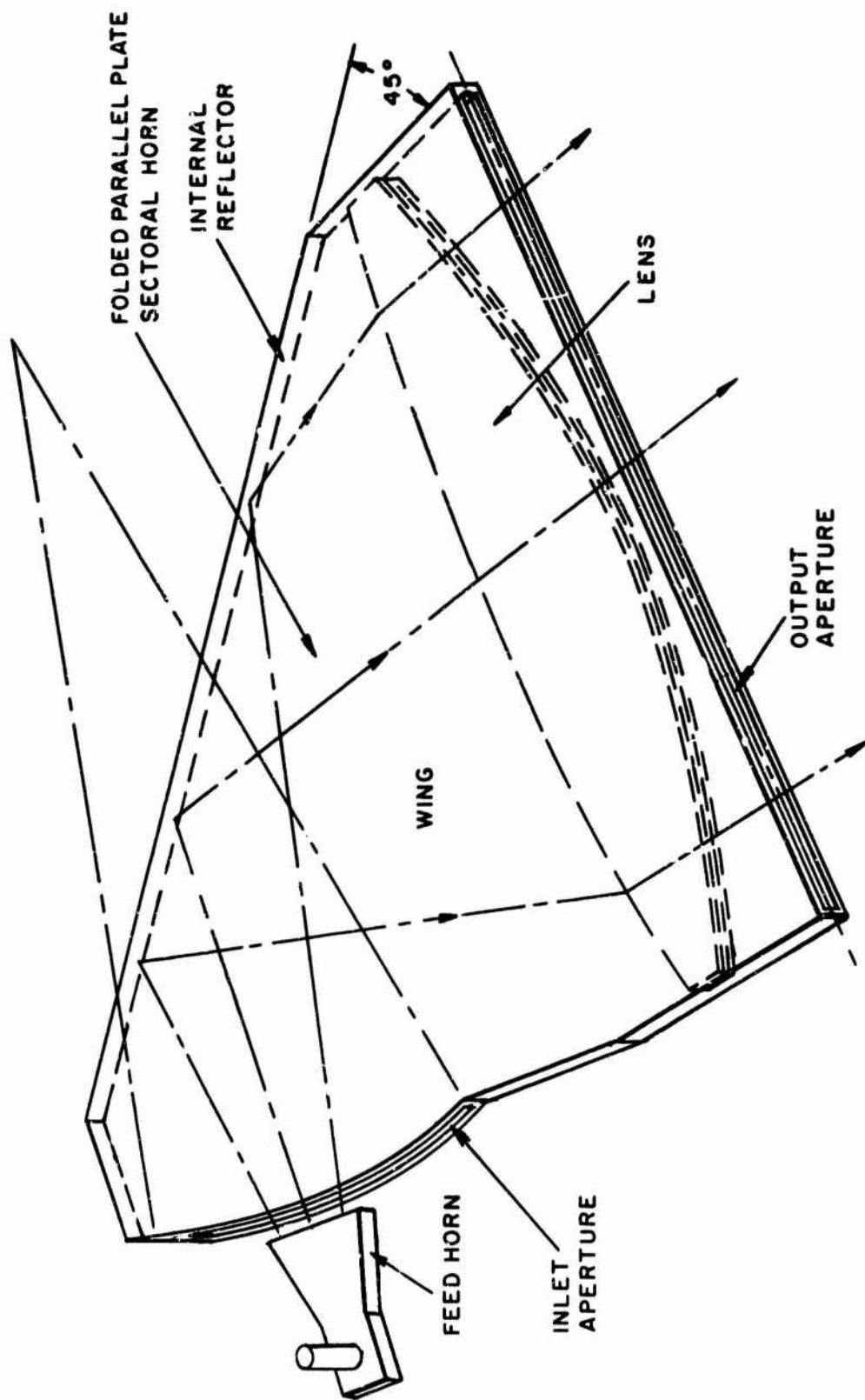


Figure 2. Lewis antenna—flat-plate layout (from U. S. Pat. 2,585,562, W. D. Lewis).

therefore, does not alter the optical (geodesic) path lengths from feed center to output lens. Also, the directions of the rays at the output relative to the original input are preserved. Therefore, the output lens design is not affected by the cone rollup. Except for the area occupied by the lens, the "wing" may be rolled in conic or cylindrical bends to achieve a compact assembly without defocusing if the bends are not excessively sharp. Figure 3 illustrates a developed Lewis antenna.

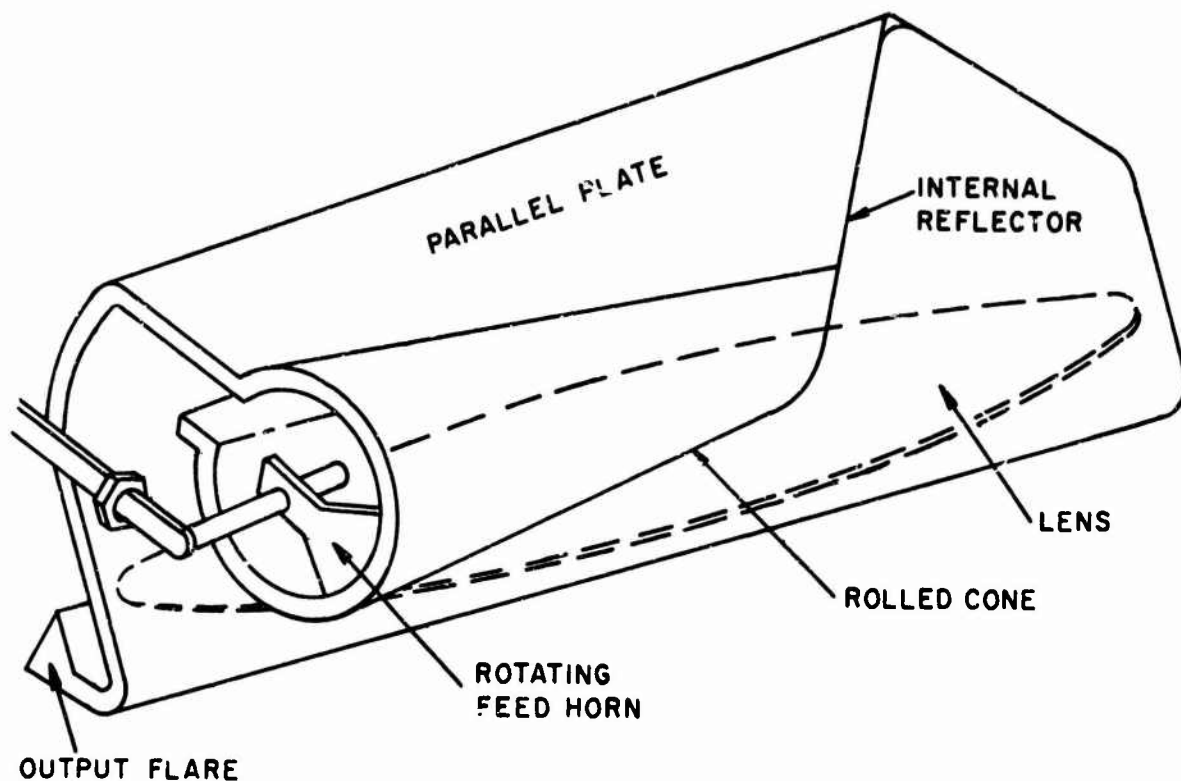
Several types of lenses have been used in these antennas—geodesic, metal plate, and dielectric. The geodesic lens is sometimes preferred because its performance can be made essentially independent of the variation of the operating frequency. An output flare is often used at the end of the trapezoidal horn to feed the output energy onto an external parabolic cylinder reflector. This reflector collimates the radiation in the plane of the beam parallel to the direction of the E-vector to form the broad dimension of the fan beam. Some performance values considered typical of this antenna are:

- |                       |  |
|-----------------------|--|
| (1) Fan beam          | 1 by 10 deg or 2 by 20 deg, 3 dB down  |
| (2) Gain at peak      | 20 to 30 dB  |
| (3) Side lobes        | 18 to 30 dB down   |
| (4) Scan sector angle | 10 to 20 deg   |
| (5) Scan rate         | 30 Hz or less  |
| (6) Peak power        | up to 2 MW, depending on microwave frequency, pulse-repetition frequency, etc. |

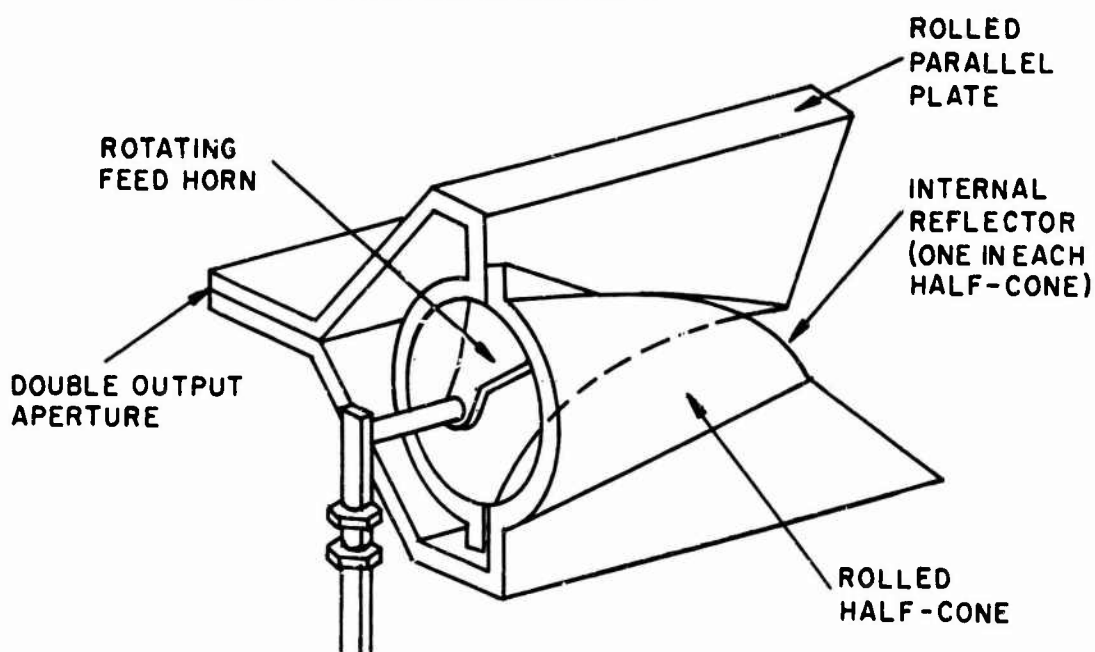
Conditions at the output aperture during transmission of energy are shown hypothetically on figure 4. Note the idealized parallel rays and corresponding flat wavefronts in the upper view of the wide dimension of the sectoral horn. The narrow beamwidth is developed in this plane in the far field, whereas the narrow aperture (bottom view, fig. 4) forms a broad beam in the far-field region.

#### 4.2.2 Cassegrainian and Schwarzschild Antennas

Cassegrainian antennas employ the reflection techniques of the Cassegrainian telescope. The antenna feed is inserted at the opening in the reflector where the eyepiece is shown in figure 1(c). The telescopic reflecting surfaces are conic sections—for example, a parabola-hyperbola combination. As can be seen in the figure, there is little room for scanning action in the 3-D telescope. This problem is solved in the Schwarzschild parallel-plate antenna by folding transversely and stretching the parallel plates at two locations



(a) ROLLED CONE SCANNER



(b) DOUBLE HALF-CONE SCANNER

Figure 3. Developed Lewis antennas (from U. S. Pat 2,585,562, W. D. Lewis).





along the optical axis so that geodesic bulges (fig. 5) or lenses are formed, which are counterparts of the telescope reflectors. Scan angles of 10 or 15 deg are possible in this design. To achieve minimum aberration, special equations have been developed, giving the bulge coordinates by (1) a Taylor series and (2) a polynomial that is a least-squares fit to "a finite number of surface points computed from the exact equations."<sup>1,2</sup> Coma is a primary consideration in the Cassegrainian and Schwarzschild designs; attempts have been made to minimize this effect.<sup>2</sup>

#### 4.3 Oblique Double-Reflector Layout

The oblique double-reflector antenna, a novel design described in detail in section 4.6, appears to be feasible in "half-Cassegrainian" counterpart.<sup>3</sup> Figure 1(c) illustrates this type antenna. If the upper half of the section view of the Cassegrainian telescope is retained and the lower half eliminated, an "oblique" design results, having its 2-D parallel-plate antenna counterpart (discussed and illustrated in sect 4.6). Conic-section reflectors are retained. Further exploration of this design will determine the correctness when the inlet aperture arc is developed into the desired circular annulus for nonoscillatory feed-motion scanning.

#### 4.4 Geodesic Lenses

##### 4.4.1 Surface-of-Revolution Lenses

Geodesic lenses may be described as those that provide focusing of the electromagnetic energy as a result of propagation over a compound curvature of the parallel conducting plates of the antenna. Geodesic lens surfaces of revolution may be generated by rotating an appropriate curved line segment about a line axis lying in the plane of the curve. There are two distinct methods of employing surface-of-revolution lenses: (1) by installing the moveable feed so that its origin, or the focal point, effectively rotates about the same axis as the surface-of-revolution axis, and (2) by installing the feed so that its motion is about an axis orthogonal to the axis of revolution.

Coma is the tendency for different zones of a lens to magnify images by different amounts. For a single off-axis object point, multiple images are formed, which are displaced from the optical axis by different amounts.<sup>4</sup>

---

<sup>1</sup>Cady et al, op cit.

<sup>2</sup>"Scanning Characteristics of Two-Reflector Antenna Systems," Airborne Inst Lab, Cutler Hammer Report, W. D. White and L. K. DeSize, Contract AF 30 (602) 1980.

<sup>3</sup>Laboratory Notebook HDL-457, 9 Dec 1969, pp. 66, 72.

<sup>4</sup>Leo Levi, op. cit.

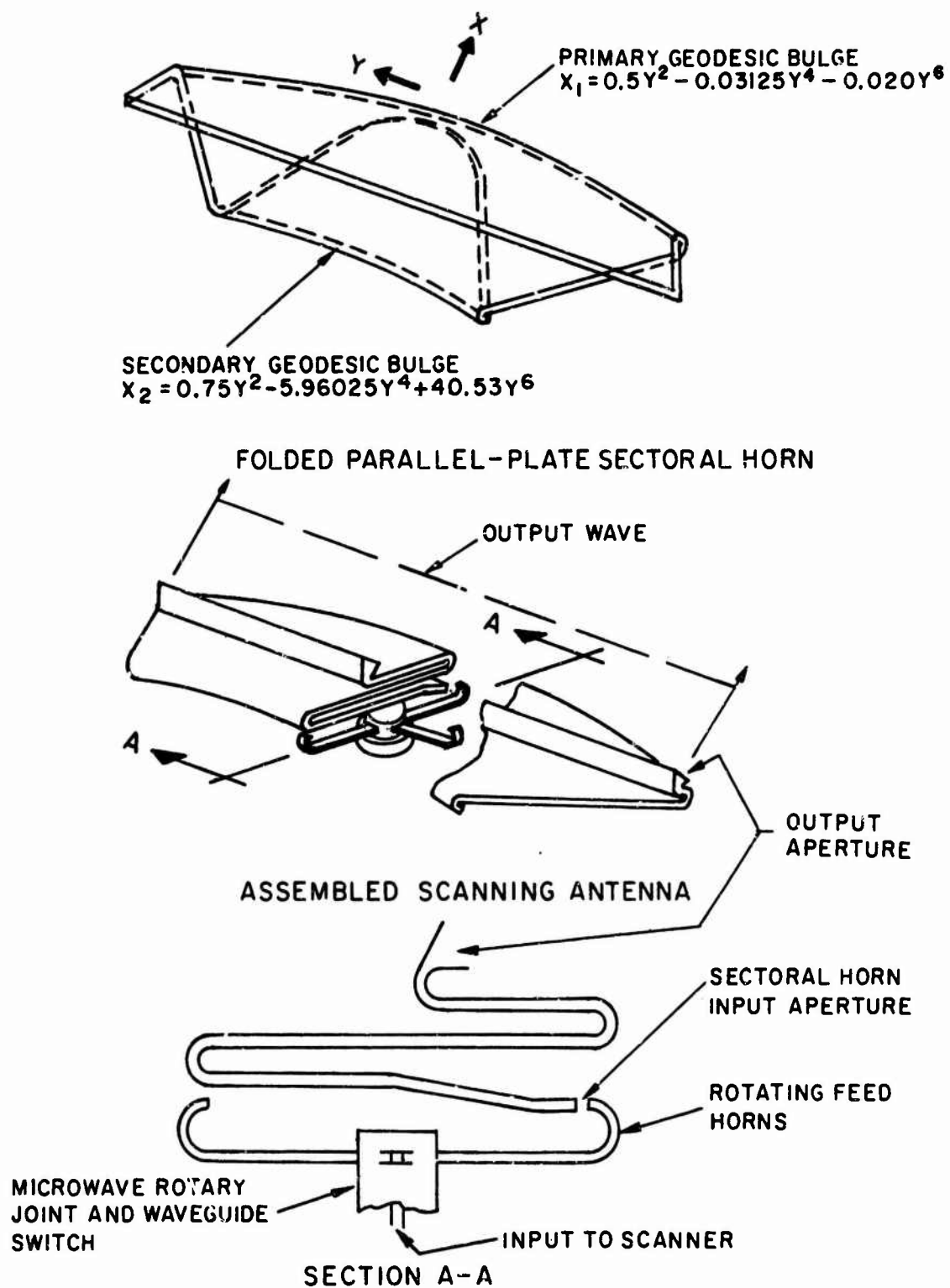


Figure 5. Schwarzschild antenna.

When the antenna lens symmetry can provide identical optical paths for all positions of the feed horn during scanning there is no coma—that is, no coma occurs when the feed horn is rotated about the same axis as that from which the lens is generated as in method (1) above. In this case, there is essentially no preferred optical axis. One lens of this type, known as the Rinehart<sup>1</sup> parallel-plate analog of the Luneburg<sup>2</sup> lens, is illustrated in figure 6. Early work was done on this lens at the Case Institute of Technology (CIT) and by RCA, Canada.<sup>3</sup>

In 1951, the Case Institute of Technology (Kunz) proposed a Rinehart-Luneburg lens having a concentric-circle ridged shape, resulting in a reduction of the lens height. Such a lens was later designed and constructed by the Georgia Institute of Technology for use on the HDL millimeter-wave radar.<sup>4</sup>

The Rinehart-Luneburg lens also has the advantage of providing large scan angles, up to 360 deg (if the feed is rotated completely around the perimeter of the lens), but usually somewhat less, such as when an output flare is provided. Some success has been reported in providing such large angles.<sup>5</sup>

The actual shape of the geodesic Luneburg surface is described by Warren-Pinnell and Johnson.<sup>6</sup> Spherical aberration is generally a consideration in these lenses.<sup>8,7</sup>

As stated, some surface-of-revolution parallel-plate lenses have coma aberration. This is the case in designs where the feed-horn scanning motion does not effectively occur about the axis of symmetry of the surface of revolution, and, consequently, when the feed horn is permitted to move off the optical axis. An example of such a lens is the semiparaboloid of revolution shown in figure 7. The phase of the outgoing wavefront is approximately linear due to the parabolic shape of the lens.<sup>8</sup>

<sup>1</sup>"Final Report—Experimental Phase," CIT, by K. S. Kunz et al, 30 Sep 1959, contract W28-099-ac-141.

<sup>2</sup>"Mathematical Theory of Optics," by R. K. Luneburg, U. of Calif, Press, 1964.

<sup>3</sup>"The Tin Hat Scanning Antenna," RCA, Canada, Prt 6, F. Warren & S. Pinnell, 16 July 1951

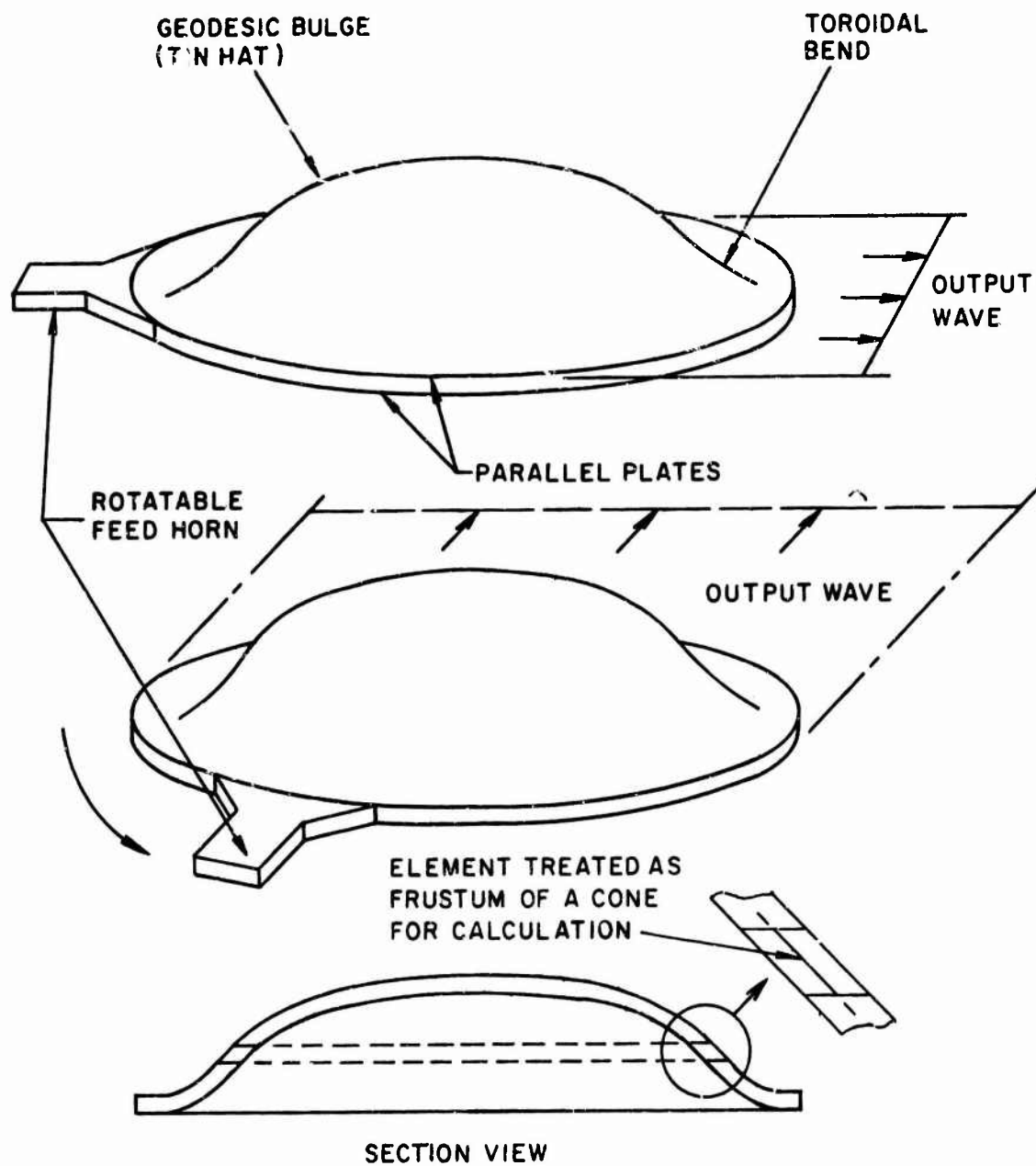
<sup>4</sup>"Millimeter-Wave Radar," Report, Georgia Inst Tech., R. M. Goodman, Jr. et al, contract AMC-275(A), 29 Feb 1968 (Conf report).

<sup>5</sup> F. Warren and S. Pinnell, op. cit.

<sup>6</sup> R. C. Johnson, op. cit.

<sup>7</sup> Ibid.

<sup>8</sup>"Microwave Antennas," A. Z. Fradin, op cit.



RINEHART-LUNEBURG GEODESIC LENS

Figure 6. Rinehart-Luneburg geodesic lens.

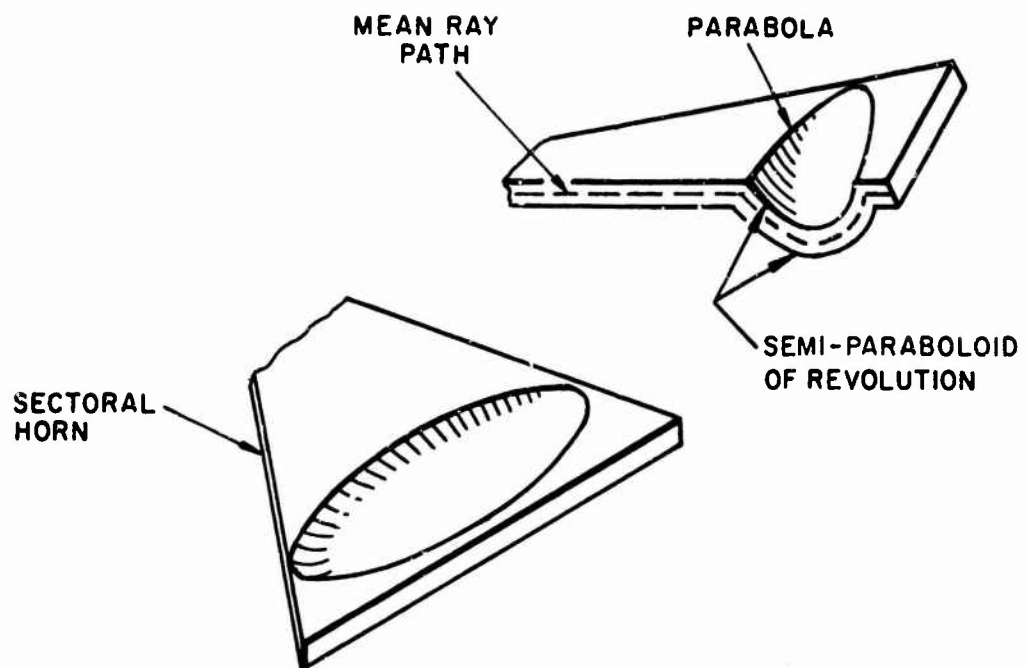


Figure 7. Semiparaboloid of revolution geodesic lens.

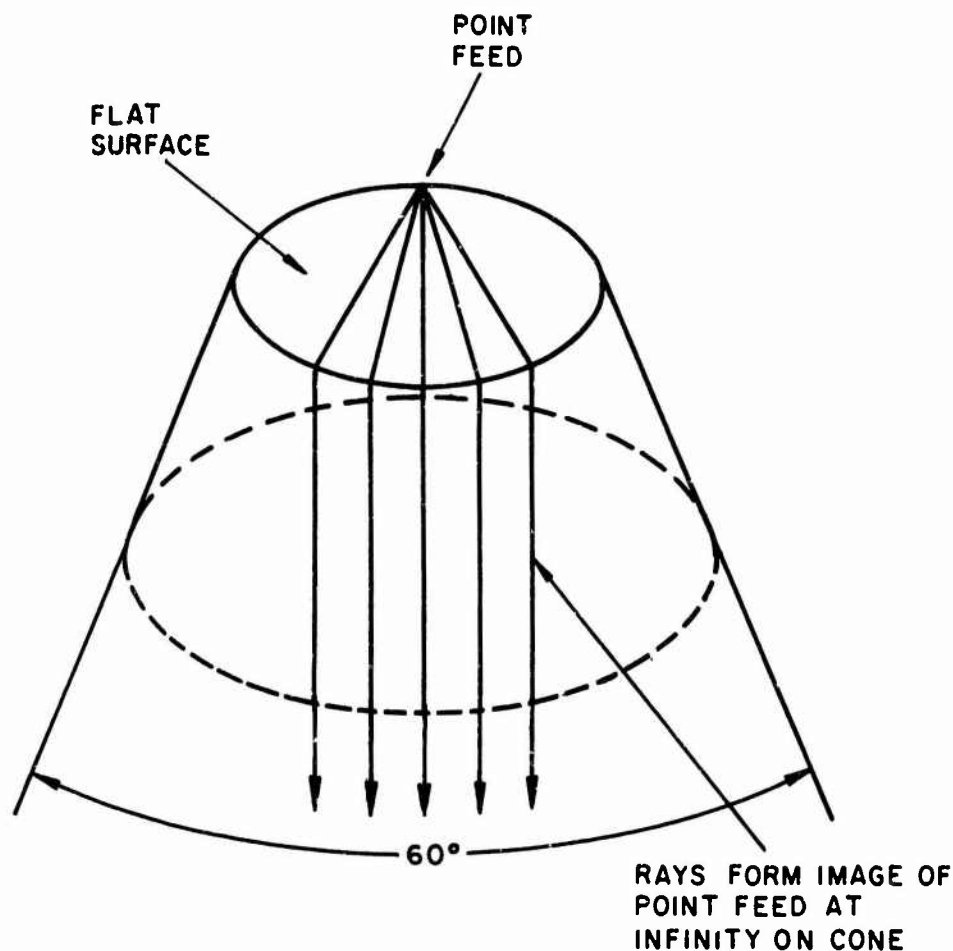


Figure 8. Lamp shade geodesic lens.

Certain geodesic lenses are formed by joining developable surfaces so that the junction of the surfaces forms a dihedral corner. The "lamp-shade" lens<sup>1</sup> is one of this type. Its median surface is that of a truncated conical frustum—a surface of revolution (fig. 8). The focal point lies on the small circle of the frustum and the output edge is a segment of the larger circle. All transmitted energy must pass the corner on the portion of the small circle opposite the feed point. This lens is said to image the point feed at infinity on the conic surface. Unfortunately, however, the parallel rays cannot lie in a linear exit aperture or a plane because of the conical shape.

<sup>1</sup>"A Family of Perfect Configuration Lenses of Revolution," *Optica Acta*, vol 1, No. 4, by G. Toraldi di Francia, Feb 1955.

The RCA R-2R lens<sup>1,2</sup> is another example of a design having the dihedral corner. This design is constructed by laying out two flat portions of a sectoral horn and joining by "warping"—not stretching—the flat plates to form a dihedral curvilinear intersection (fig. 9). This does not result in a surface of revolution because the output edge is made linear. Except for the linear output edge, however, the geometry is basically similar to that of the lamp-shade lens.

#### 4.4.2 Other Lens Configurations

There are numerous other geodesic lens configurations that are not surfaces of revolution. One example is the parabolic bulge lens in figure 10. The leading edge of the folded-over, parallel-plate region is approximately a parabolic shape, resulting in flattening of an outgoing wave at the exit aperture. Another configuration that would provide phase equalization of an outgoing wave is represented by the copper-wire model, figure 11. The median surface of the parallel-plate region and the ray paths are approximated in the model by the wires. Circular waves emanating from a feed point are depicted as becoming linear after traversing the lens hump.

#### 4.5 Internal Reflectors

Linear or "linear-developable," curved reflectors are used frequently in parallel-plate antennas to achieve the desired compact foldup or rollup—such as in Lewis, Foster, and R-2R antennas (linear-developable, curved reflectors are those that would become linear if the median surface were unrolled). Recent work at HDL and at other installations now tends to confirm that (1) a planar reflector may be installed in any surface-of-revolution lens, corresponding to method (1) described in sect 4.4.1, passing through the center of symmetry, generally resulting in an equivalent lens of one-half the original volume; and (2) curvilinear reflectors can be installed in the flat, parallel-plate, sectoral horn to perform the focusing normally done

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<sup>1</sup>A. Z. Fradin, op. cit.

<sup>2</sup>RCA Review, vol IX, Dec 1948.

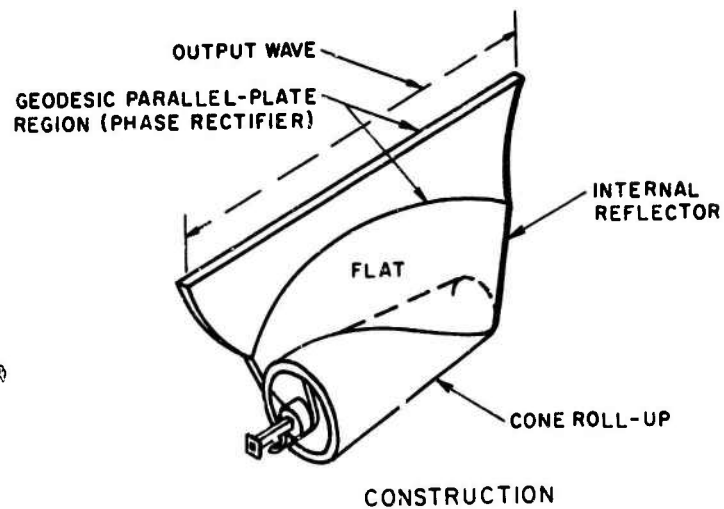
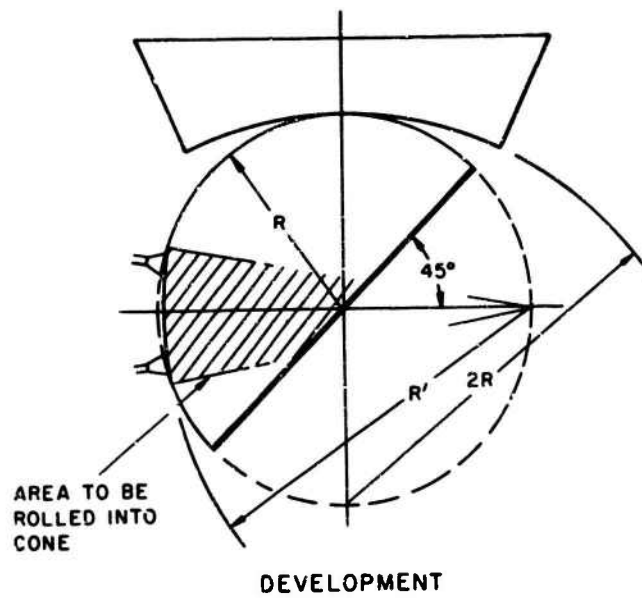


Figure 9. R-2R antenna.

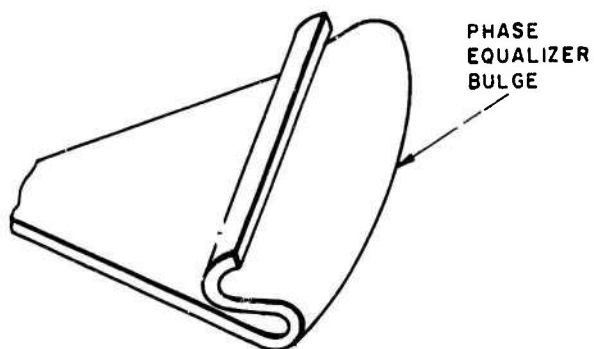
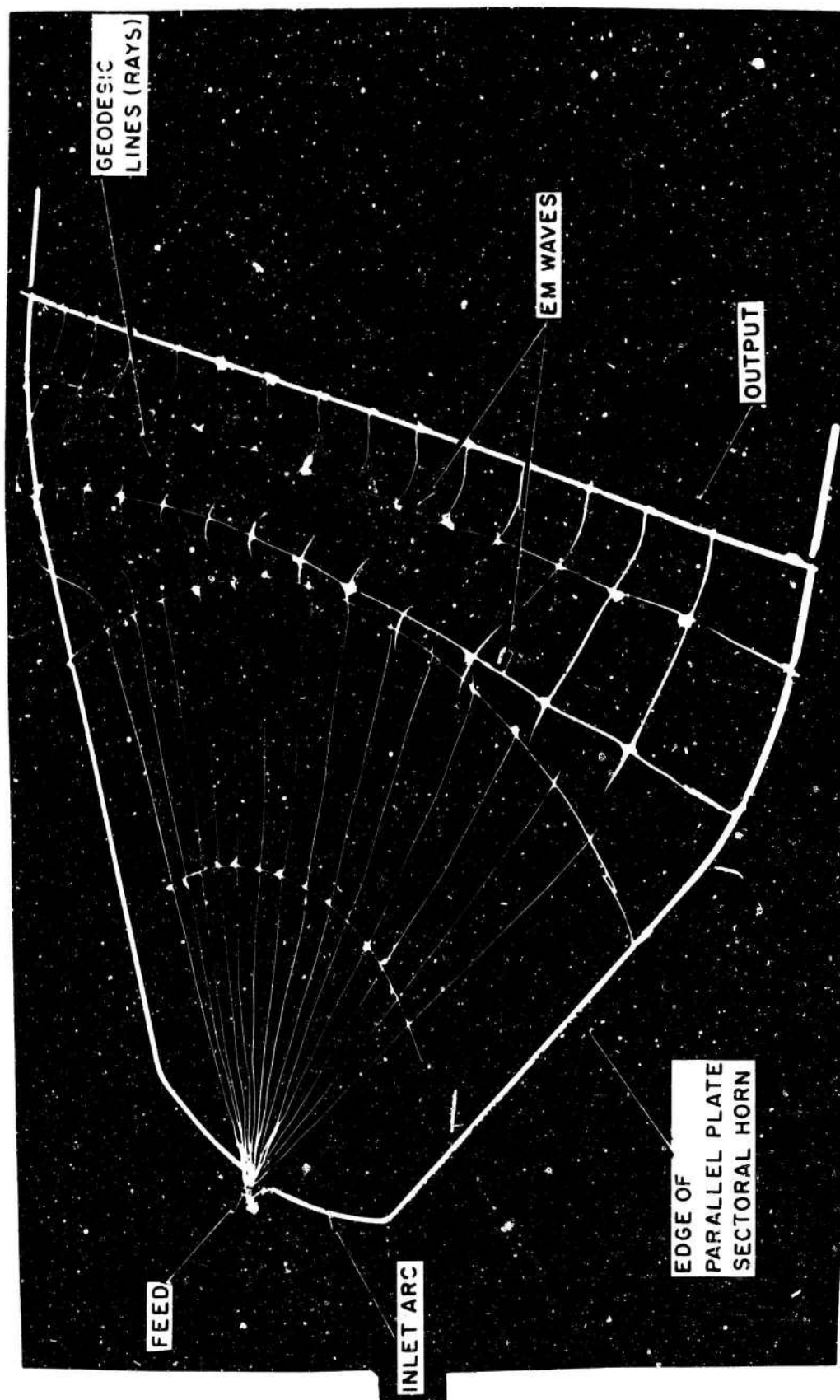


Figure 10. Parabolic bulge geodesic lens.





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Figure 11. Copper-wire, scale model of sectoral-horn lens.

by complicated lenses, resulting in volume reduction. Figure 12 illustrates the installation of an internal parabolic reflector in a Lewis geometry. This design may be rolled up, providing nonoscillatory scanning and further reducing antenna bulk as illustrated in figure 13.

#### 4.6 Oblique Double-Reflector Scanning Antennas

In early 1968, our investigations at HDL showed that various oblique-double-reflector, parallel-plate, scanning antennas are feasible. Some of these variations include combinations of double-linear reflectors and various geodesic, dielectric, and metal-plate lenses having circular, continuous-feed-motion scanning.<sup>1</sup> Figures 14(a) and (b) illustrate a double-linear-reflector design having a parabolic bulge geodesic output lens and a conic-development, flat-plate area to provide the input feed annulus. Several Lucite parallel-plate, small-scale models were constructed in early 1968 to verify the physical and optical geometry of the arrangement (fig. 14b).

In 1969, we considered the design of oblique-multiple-reflector, parallel-plate antennas having one or more conic-section, curved-line reflectors. Two examples of flat-plate layouts of this type, both compatible with inlet aperture rolling up for continuous-motion scanning, are shown in figures 12 and 15. In figure 12, the primary (output) reflector is in the shape of a segment of a parabola having its center-of-scan feed position coincident with the parabolic focus; the secondary reflector is linear. The same design, but with the scanner rolled into the characteristic cone shape, is depicted in figure 13. This layout has essentially the same configuration as that of the scale models with lenses constructed in 1968. There is little doubt about the possibility of constructing a workable full-scale model. Some areas that should be investigated include (1) what correction, if any, that should be applied to the antenna to offset the tendency for the output illumination to be unsymmetrical, and (2) how the off-scan-center coma compares with that of other small scan-angle antennas. It may be possible with an asymmetrical scanning feed horn to compensate for or rectify the unsymmetrical output illumination, if necessary. Theoretically, the parabolic reflector antenna is perfectly free of spherical aberration which afflicts geodesic lenses.

Figure 15 depicts a flat-plate layout of the oblique-double reflector design having both reflectors of conic section line segments—a parabola and a hyperbola. The parabolic focal point is imaged by the hyperbolic reflector so that the real image is positioned off the focal axis of the parabola and in a location closer to the parabolic segment, tending to reduce the antenna volume. If we assume now that the scan inlet arc, the parabolic segment, and the associated parallel

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<sup>1</sup>Laboratory Workbook HDL-457, op. cit.

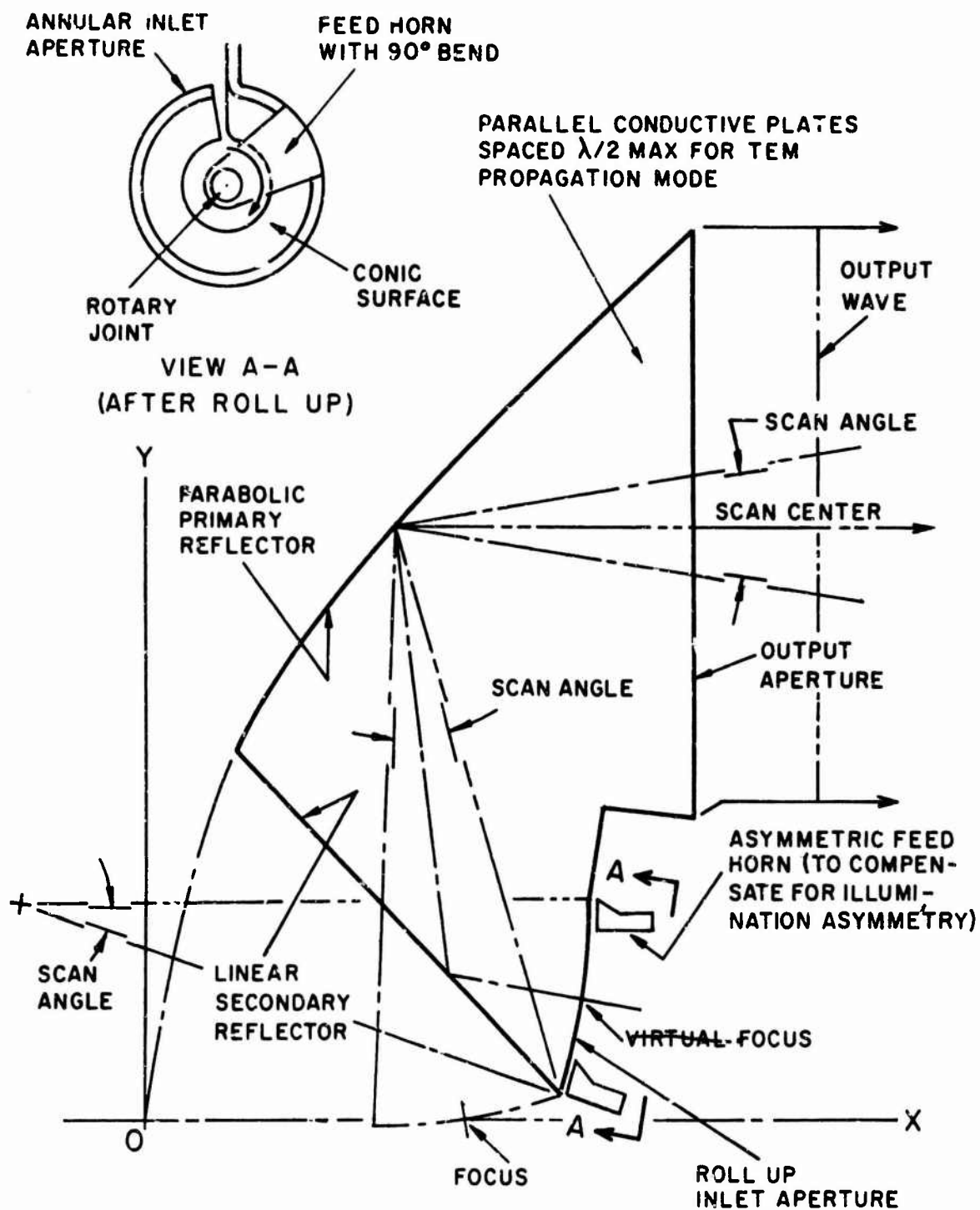


Figure 12. Parallel-plate antenna having linear and parabolic line reflectors (oblique double reflector antenna).

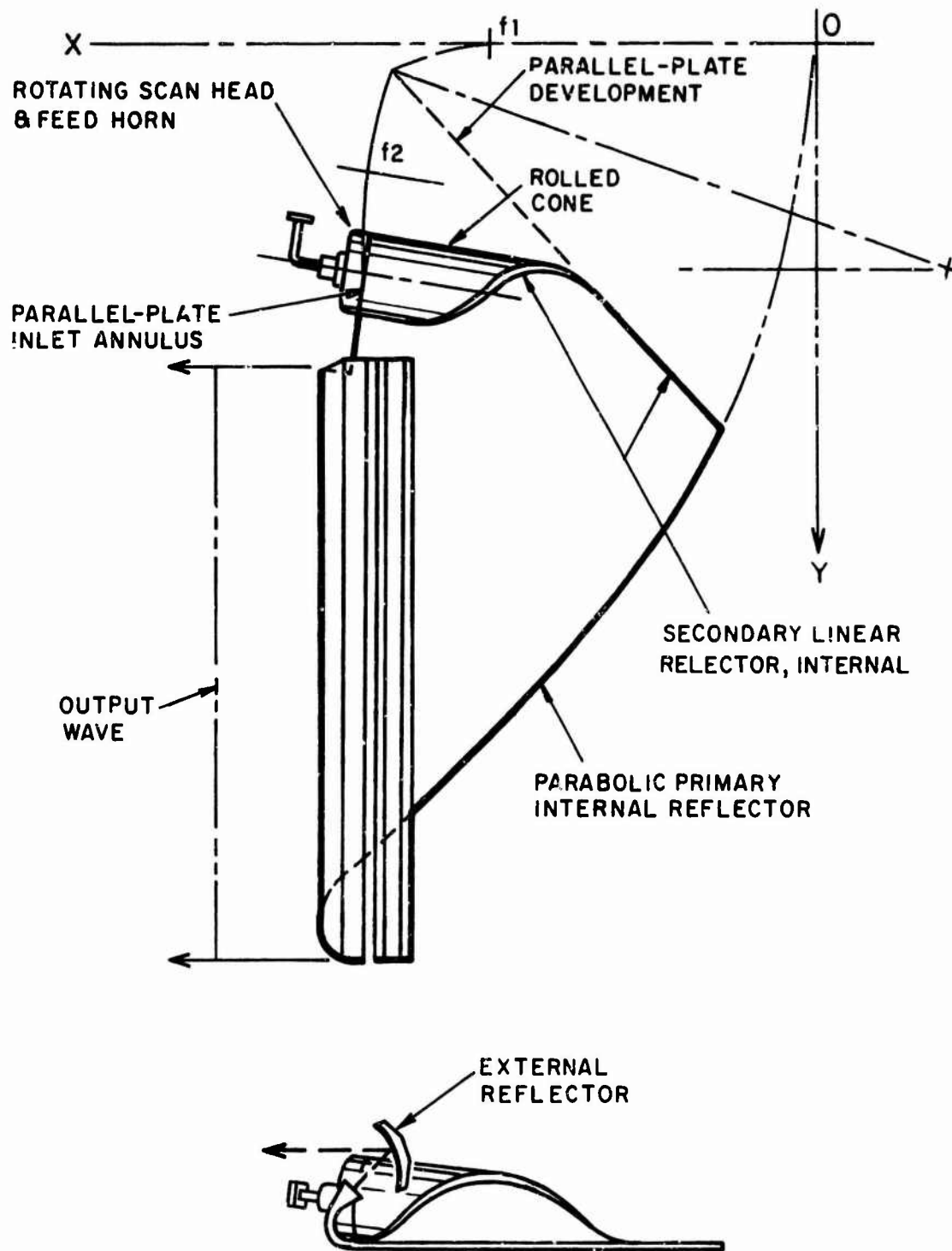


Figure 13. Rolled parallel-plate antenna having linear and parabolic line reflectors.

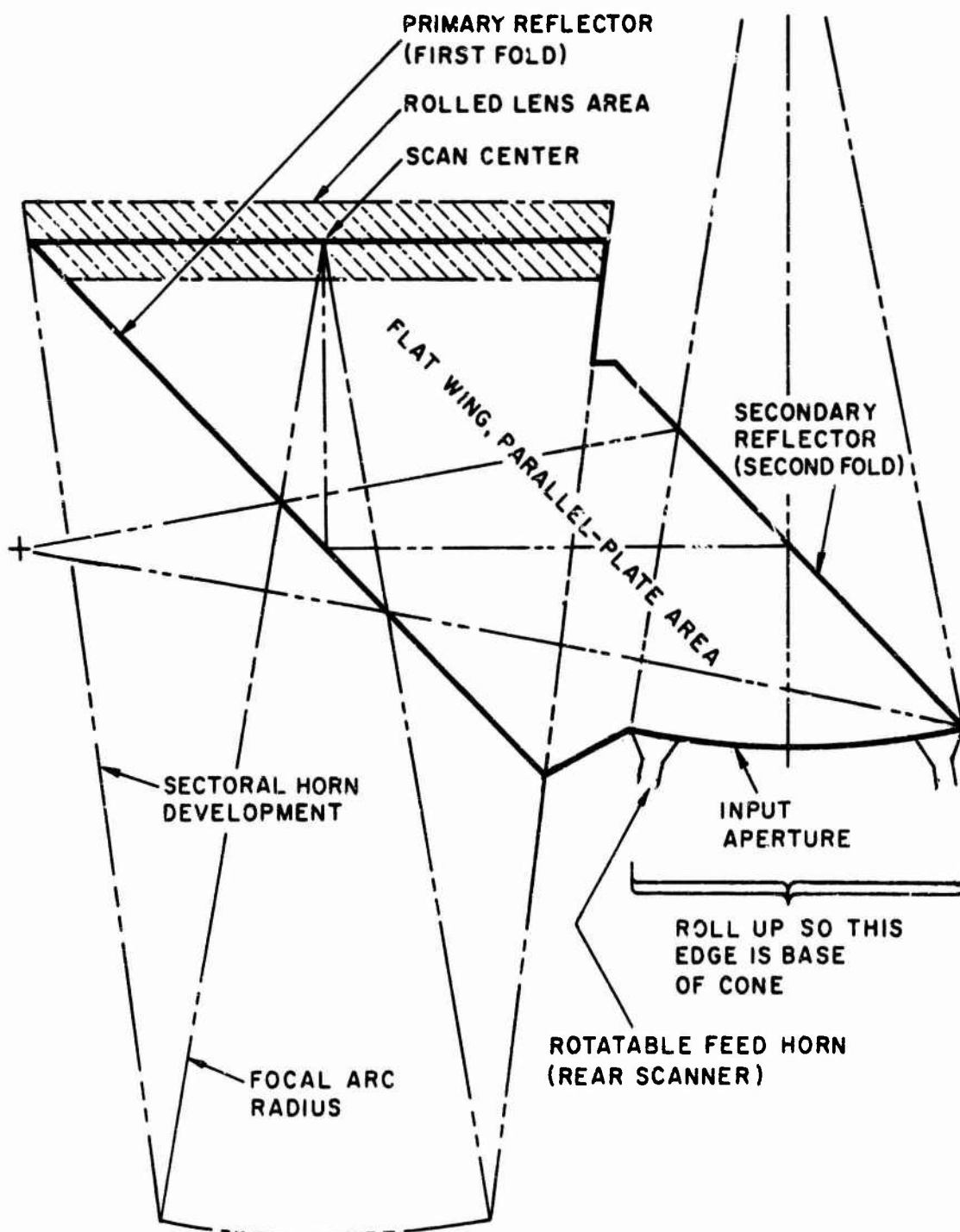


Figure 14(a). Oblique, double-reflector, parallel-plate antenna with rear scanner.

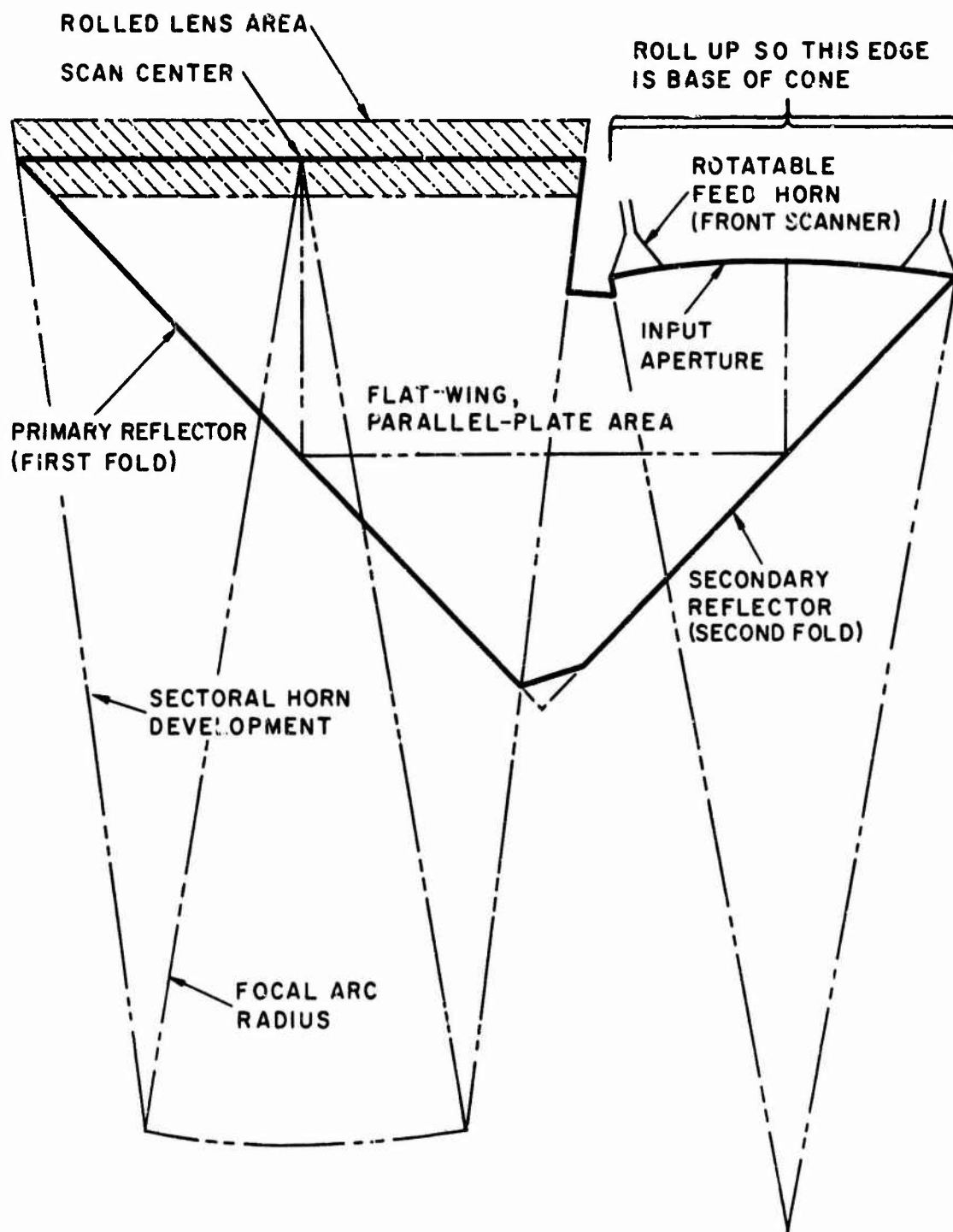


Figure 14(b). Oblique, double-reflector, parallel-plate antenna with front scanner.

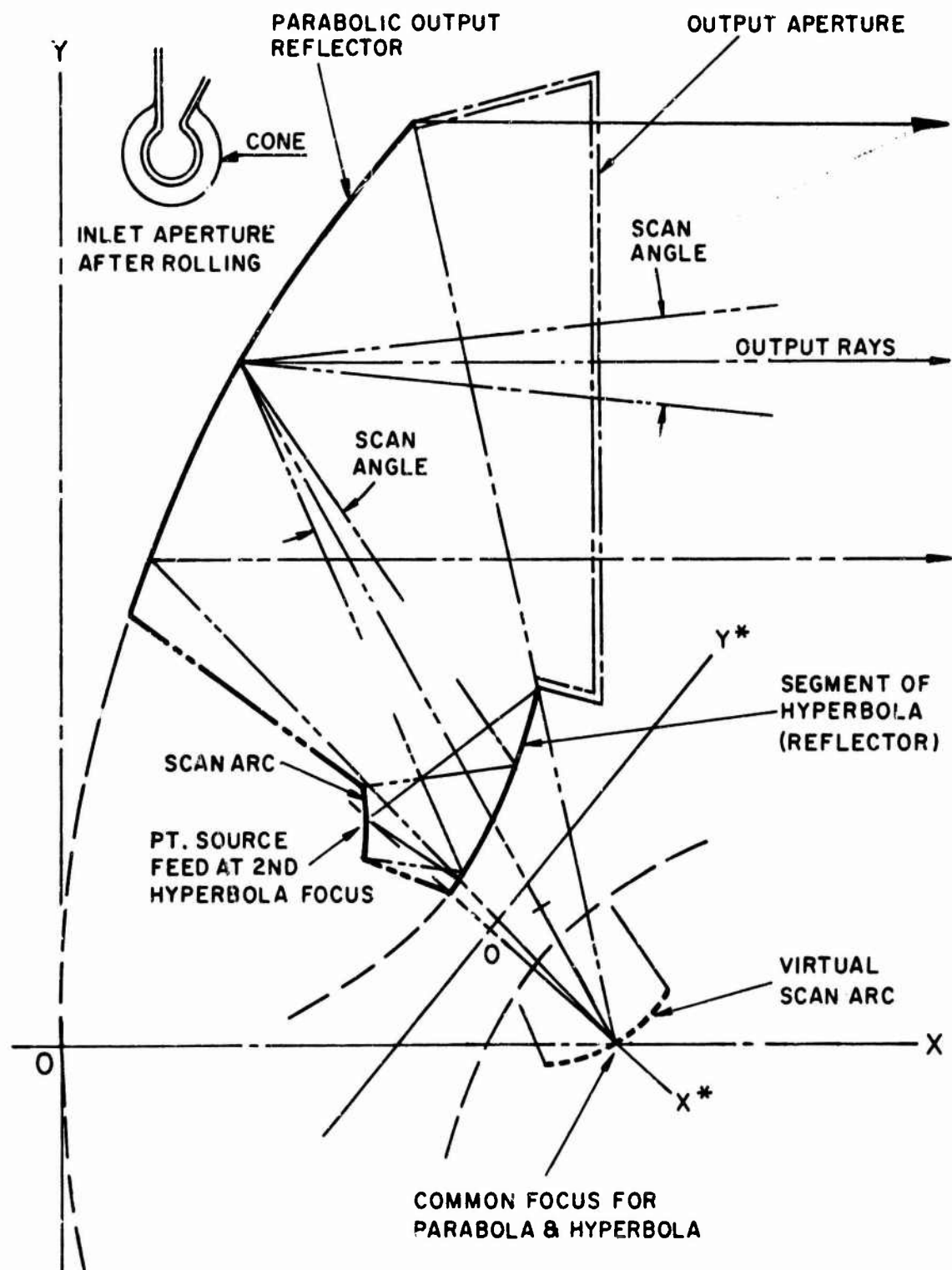


Figure 15. "Half-Cassegrainian," two-reflector, parallel-plate antenna (oblique, double-conic reflector).

plates are rolled—for example, into a cone—a circular aperture may be obtained, providing a basis for continuous-motion scanning, a smaller diameter scanner, a compact antenna, and absence of spherical aberration in a single antenna.

These designs may be referred to as "half-Cassegrainian" because of the conic-section-curve reflectors employed and the similarity of the oblique reflection to one-half the Cassegrainian telescope layout.

## 5. CONCLUSIONS

The results of this effort show that various compact and useful parallel-plate antenna geometries exist that were not fully realized heretofore. These geometries, which are compatible with the Lewis principle of continuous-motion scanning, are the result of judicious rolling, folding, and/or stretching of the hypothetical parallel-plate median surfaces. In some respects, the formation of multiple EM-wave reflections within the antenna is analogous to optical reflections in telescopes. Geodesic lenses and microwave reflectors can be treated by the same basic laws of geometrical optics as are applied to optical lenses and reflectors.

Linear and curvilinear reflectors of both single and multiple oblique designs may be employed as can geodesic lens reflectors. Novel combinations of these elements, some of which can be expected to provide improved performance, are described herein. Other combinations are being investigated and others undoubtedly remain to be discovered. For certain applications, it is virtually a certainty that further investigations will result in the development of radar systems that have improved electrical performance and are more mobile and less costly. The present emphasis being placed on the development of electronically scanned phased-array antennas may tend to obscure the relative ease of scanning or steering megawatt radiation lobes by electrical scanning techniques, requiring only tens or hundreds of watts of scanner drive power.

In addition, hybrid systems combining some of the features of phased arrays, such as multiple dipole element output apertures, and some of the features of the parallel-plate nonoscillatory scanner are now being developed by other researchers. Such antennas should be capable of high-power operation without the critical microwave switching problems sometimes encountered.

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13. ABSTRACT Design configurations of nonoscillatory feed, line-scan, parallel-plate radar antennas have been explored. Some historically important designs are described as a basis for understanding several new, proposed antenna geometries. It is shown that the continuous-motion scanner developed by Lewis can be combined with many parallel-plate lenses and reflectors in various geometric layouts to obtain useful microwave antennas. Designs that would normally produce good radiation by combining the continuous-motion scanner and the cut, rolled, and folded sectoral horn-lens assemblies.  The continuous-motion scanner can be combined with dielectric, metal-plate, and geodesic lenses. Multiple linear and nonlinear reflectors may be installed in the sectoral horn and planar reflectors may be used in some geodesic lenses to achieve compactness. Some of these "oblique" layouts are shown to be analogous to optical telescopes. Methods of reducing antenna size by folding over or rolling up flat portions of the sectoral horn are explained.  Optical laws are shown to apply to the geometrical design of parallel-plate antennas and lenses. Coma and spherical aberration are defined and procedures for designing parallel-plate radar antennas are proposed. Choice of optical and radar system parameters and the interrelationship of such parameters as scan angle, focal length, beamwidth, scan rate, and physical dimensions are discussed. Fermat's extremum principle is defined and some fundamental criteria for designing geodesic lenses are outlined. Also described in this report are the oblique, multiple-reflector antenna designs conceived at HDL by the author. The versatility and compactness of these designs when combined with the		

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Non-oscillatory feed motion scanners	8	3				
Electrical and electromechanical radar scanners	8	3				
Geometrical optics	8	2				
Microwave optics	8	2				
Geodesic microwave lenses and reflectors	8	3				